National Nonpoint Source Monitoring Program

Tech NOTES 7

December 2011

Jean Spooner, Steven A. Dressing, and Donald W. Meals. 2011. Minimum detectable change analysis. Tech Notes 7, December 2011. Developed for U.S. Environmental Protection Agency by Tetra Tech, Inc., Fairfax, VA, 21 p. Available online at *www.bae.ncsu.edu/programs/extension/wqg/319monitoring/tech_notes.htm*.

Through the National Nonpoint Source Monitoring Program **(NNPSMP)**, states monitor and evaluate a subset of watershed projects funded by the Clean Water Act Section 319 Nonpoint Source Control Program.

The program has two major objectives:

1. To scientifically evaluate the effectiveness of watershed technologies designed to control nonpoint source pollution
2. To improve our understanding of nonpoint source pollution

NNPSMP Tech Notes is a series of publications that shares this unique research and monitoring effort. It offers guidance on data collection, implementation of pollution control technologies, and monitoring design, as well as case studies that illustrate principles in action.

Minimum Detectable Change Analysis

Introduction

The purpose of this technical note is to present and demonstrate the basic approach to minimum detectable change (MDC) analysis. This publication is targeted toward persons involved in watershed nonpoint source monitoring and evaluation projects such as those in the National Nonpoint Source Monitoring Program (NNPSMP) and the Mississippi River Basin Initiative, where documentation of water quality response to the implementation of management measures is the objective. The MDC techniques discussed below are applicable to water quality monitoring data collected under a range of monitoring designs including single fixed stations and paired watersheds. MDC analysis can be performed on datasets that include either pre- and post-implementation data or just the typically limited pre-implementation data that watershed projects have in the planning phase. Better datasets, however, provide more useful and accurate estimates of MDC.

Minimum detectable change analysis can answer questions like: "How much change must be measured in a water resource to be considered statistically significant?"

or

"Is the proposed monitoring plan sufficient to detect the change in concentration expected from BMP implementation?"

Minimum Detectable Change

The Minimum Detectable Change (MDC) is the minimum change in a pollutant concentration (or load) over a given period of time required to be considered statistically significant.

The calculation of MDC has several practical uses. Data collected in the first several years of a project or from a similar project can be used to determine how much change must be measured in the water resource to be considered statistically significant and not an artifact of system variability. Calculation of MDC provides feedback to the project managers as to whether the proposed land treatment and water quality monitoring designs are sufficient to accomplish and detect the expected changes in water quality over a pre-specified length of time. These calculations facilitate realistic expectations when evaluating watershed studies. Calculation of the magnitude of the water quality change required can serve as a useful tool to evaluate water quality monitoring designs for their effectiveness in detecting changes in water quality. Closely related, these calculations can also be used to design effective water quality monitoring networks (Spooner et al., 1987; 1988).

Bernstein and Zalinski (1983) make a valid distinction between the magnitude of the 'statistically' and 'biologically' significant changes. The size of a statistically significant detectable change depends on the number of samples. For a fixed sample variability, a large number of samples results in a large number of degrees of freedom in the statistical trend test, and therefore, a relatively small value for the MDC. However, a small statistically significant difference may have no biological or practical significance. In contrast, with small sample sizes, statistically significant detectable changes may be much larger than biologically significant changes. A system may have exhibited a biologically significant change that cannot be statistically detected because sample sizes are too small.

MDC is an extension of the Least Significant Difference (LSD) concept (Snedecor and Cochran, 1967). The MDC for a system can be estimated from data collected within the same system or similar systems. A system is defined by the watershed size, water resource, monitoring design, pollutants measured, sampling frequency, length of monitoring time, hydrology, and meteorology.

MDC is a quantity that is calculated using the pre-planned statistical trend tests on the measured observations, typically in the pre-BMP project phase. MDC is used as a guide to calculate the minimum amount of change expected to be detected given the sample variability, number of samples, monitoring design, statistical trend tests, and significance level.

MDC analysis must be consistent with and based on the planned statistical approach to analyzing project data.

General Considerations

The following assumptions are made in the calculation of MDC.

- Historical sample measurements are representative of the temporal and spatial variation of the past and future conditions.
- Variability due to sampling, transport or laboratory error is negligible compared to variability over time.

Typically, the pollutant concentrations or load values exhibit a log-normal distribution. When this is the case, the MDC is expressed as a percent change relative to the initial annual geometric mean concentration. Given a particular monitoring scheme, the water quality observations and their variability can be used to calculate the MDC required in the geometric mean pollutant concentration over time.

When the water quality values are log-normal, calculations for the MDC values are performed on the base 10 logarithmic scale. Analyses on the logarithmic scale have several beneficial features:

- The log normal distribution generally fits the distribution of water quality data. One feature of a log normal distribution is skewed data on the original scale (e.g., many lower values with a few higher values).
- The logarithmic transformation on the water quality variables is usually required for the distributional assumptions of parametric trend analyses to be met.
- The results become dimensionless and are independent of the units of measurements.
- MDC can be expressed as a percentage, rather than an absolute difference, because the calculations are performed on the logarithmic scale.

Sampling frequency determination is very closely related to MDC calculations. Sample size determination is usually performed by fixing a significance level, power of the test, the minimum change one wants to detect, the duration of monitoring, and the type of statistical test. MDC is calculated similarly except the sample size (i.e., number of samples) is fixed and the power is set to 50 percent. MDC is the amount of change you can detect given the sample variability. Many of the formulas that are used for confidence limit and sample size determination are similar to those used to calculate MDC.

Sampling frequency and MDC are closely related parameters. The planned sampling frequency and duration strongly influence the MDC, and the MDC largely dictates the sampling frequency necessary to measure such change within a specified time period.

Factors Affecting the Magnitude of the MDC

The MDC is a function of pollutant variability, sampling frequency, length of monitoring time, explanatory variables or covariates (e.g., season, meteorological, and hydrologic variables) used in the analyses which 'adjust' or 'explain' some of the variability in the measured data, magnitude and structure of the autocorrelation, and statistical techniques and the significance level used to analyze the data.

Spatial and Temporal Variability

The basic concept in the calculation of MDC is simple: variability in water quality measurements is examined to estimate the magnitude of changes in water quality needed to detect significant differences over time. Hydrologic systems are highly variable, often resulting in large values for MDC. Variations in water quality measurements occur in both spatial and temporal dimensions, and are due to several factors including:

- A change in land treatment resulting in decreased concentrations and/or loadings to receiving waters (determining the amount of water quality change is usually a key objective of a watershed project)

- Sampling and analytical error
- Monitoring design (e.g., sampling frequency, sampling location, variables measured)
- Changes in meteorological and hydrologic conditions
- Seasonality
- Changes in input to and exports from the system. For example, changes in upstream concentrations can affect the downstream water quality.

MDC is proportionally related to the standard deviation of the sample estimate of trend (e.g., standard deviation of the sample estimate of slope for a linear trend or standard deviation of samples in the pre-BMP time period for a step trend). This standard deviation is a function of the variability in Y that is not explained by the statistical trend model (i.e., error variance). As such, any known sources of variation that can be added to the statistical trend model to minimize the error variance will also serve to reduce the MDC and increase the ability to detect a real change in water quality due to land treatment. For example, adjusting for changes in explanatory variables such as streamflow or changes in land use (other than the BMPs) would reduce both the standard error and the MDC.

It should be noted that sample variability may be affected by sampling frequency. For frequent sampling directed at including storm events, variability is usually higher than for fixed-interval sampling directed at monitoring ambient conditions. In addition, the nature of collection and data aggregation will directly affect the variability and the autocorrelation. Composite or aggregated samples are generally less variable than single grab samples and exhibit a lower degree of autocorrelation as compared to non-aggregated data.

Sampling Frequency and Record Length

The MDC calculation is the change required for a specified sample frequency and duration. MDC decreases with an increase in the number of samples and/or duration of sampling.

Increasing sampling frequency and/or record length (e.g., increasing the number of years for monitoring) results in an increase in the number of samples (N), and therefore increases the degrees of freedom in the statistical trend tests and results in a smaller MDC value. Increasing the number of samples results in a decrease in MDC (on the logarithmic scale) approximately proportional to the increase in the square root of N. However, increasing N by increasing the sample frequency may not decrease the MDC by this total proportion due to the effects of temporal autocorrelation.

Increasing record length has several advantages over increasing sampling frequency. Increasing record length serves to add degrees of freedom to the statistical trend models. In addition, increasing the number of years adds extra verification that the observed changes are real and not a result of an unknown or unmeasured variable that also exhibits large year-to-year variations. Increasing record length also serves to increase the time base from which extrapolations may be made.

Seasonal, Meteorological and Hydrologic Variability

The standard error of a trend estimate can effectively be reduced by accounting for seasonality and meteorological and hydrologic variables in the trend tests. Because these variables or covariates can help reduce the amount of variability that cannot be 'explained' they are commonly called 'explanatory variables.' For example, Hirsch and Gilroy (1985) found that a model that removes variability in sulfate loading rates due to precipitation and varying seasonal mean values can reduce the step trend standard deviation by 32%, and therefore, the magnitude of change needed for statistically detectable change would also be reduced by 32%.

Incorporation of appropriate explanatory variables increases the probability of detecting significant changes and serves to produce statistical trend analysis results that better represent true changes due to BMP implementation rather than changes due to hydrologic and meteorological variability. Commonly used explanatory variables for hydrologic and meteorological variability include streamflow and total precipitation.

Adjustment for seasonal, meteorological and hydrologic variability is also important to remove bias in trend estimates due to changes in these factors between sampling times and years. Interpretations regarding the direction, magnitude, and significance in water quality changes may be incorrect if hydrologic and/or meteorological variability is not accounted for in the statistical trend models.

If significant variation exists between the seasonal means and/or variances and is not considered in the statistical trend models, then the assumptions of identical and independent distribution of the residuals (from the statistical model) will be violated and the results for the statistical trend analyses (both parametric and nonparametric) will not be valid. Non-identical distributions can occur when the seasonal means vary from the overall mean and/or the variances within seasons are different for each season. Non-independence can occur because seasons have cyclic patterns, e.g., winters are similar to winters, summers to summers, etc.

Autocorrelation

Temporal autocorrelation exists if an observation is related or correlated with past observations (not independent). Autocorrelation in water quality observations taken less frequently than daily is usually positive and follows an autoregressive structure of order 1, AR(1). More complicated autocorrelation models (AutoRegressive Integrated Moving-Average or ARIMA models with more lag terms and moving average terms) are usually needed for daily or more frequent sampling designs. Positive autocorrelation usually results in a reduction of information (e.g., less degrees of freedom than the actual number of samples) in a data series and affects statistical trend analyses and their interpretations. Each additional sample adds information, but not a full degree of freedom if it's not independent of the previous sample.

If significant autocorrelation exists and is not considered in the statistical trend models, then the assumption of independence of the residuals will have been violated. The result is incorrect estimates of the standard deviations on the statistical parameters (e.g., mean, slope, step trend estimate) which in turn results in incorrect interpretations regarding the statistical significance of these statistical parameters. Autocorrelation must be incorporated into the statistical trend models to obtain an accurate estimate of MDC (e.g., using time series analyses). Autocorrelation can also be reduce by data aggregation (e.g., weekly, monthly), but this will decrease the degrees of freedom.

Statistical Trend Tests

MDC is influenced by the statistical trend test selected. For the MDC estimate to be valid, the required assumptions must be met. Independent and identically distributed residuals are requirements for both parametric and nonparametric trend tests. Normality is an additional assumption placed on most parametric trend tests. However, parametric tests for step or linear trends are fairly robust and therefore do not require 'ideally' normal data to provide valid results.

The standard error on the trend estimate, and therefore, the MDC estimate will be minimized if the form of the expected water quality trend is correctly represented in the statistical trend model. For example, if BMP implementation occurs in a short period of time after a pre-BMP period, a trend model using a step change would be appropriate. If the BMPs are implemented over a longer period of time, a linear or ramp trend would be more appropriate.

> MDC is influenced by the statistical trend test selected. The MDC will be minimized if the correct statistical trend model (e.g., step vs. linear or ramp) is selected.

A step change can be examined by the use of tests such as the parametric Student's *t*-test or the nonparametric Wilcoxon rank sum test. The two-sample Student's *t*-test and the nonparametric Wilcoxon rank sum tests for step change are popular step change tests used in water quality trend analyses because they are easy to use. Analysis of Covariance (ANCOVA) can test for step changes after adjusting for variability in explanatory variables or covariates (e.g., streamflow). When a sudden system alteration, such as BMP implementation occurs, the BMPs can be called an 'intervention.' In statistical terms, intervention analysis can be used to extend the two-sample Student's *t*-test to include adjustments for autocorrelation.

The most popular types of statistical models for linear change include the parametric linear regression and the nonparametric Kendall's tau (with the Sen's Slope Estimator). Autocorrelation is most easily accounted for by the use of linear regression models with time series errors. When using a statistical software package that can adjust for autocorrelation (e.g., PROC AUTOREG in SAS (SAS, 1999)), it requires no extra effort to correctly incorporate the needed time series as well as explanatory variables. See Tech Notes #6 (Meals et al. 2011) for an overview of other statistical software packages that may be useful here.

Steps to Calculate the MDC

The calculation MDC or the water quality concentration change required to detect significant trends requires several steps. The procedure varies slightly based upon:

- Pattern of the expected change and therefore appropriate statistical model (e.g., step, linear, or ramp trend).
- Whether the data used are in the original scale (e.g., mg/l or kg) or log-transformed.
- Incorporation of time series to adjust for autocorrelation.
- Addition of explanatory variables such as streamflow or season.

The following steps and examples are adopted from Spooner et al. (1987 and 1988):

Step 1. ***Define the Monitoring Goal and Choose the Appropriate Statistical Trend Test Approach.*** One goal may be to detect a statistically significant linear trend in the annual mean (geometric mean if using log-transformed data) pollutant concentrations that may be related to land treatment changes. A linear regression model using log-transformed data would then be appropriate. An alternative goal to detect a statistically significant change in the post-BMP period as compared to a pre-BMP period would require a step change statistical test such as the *t*-test or ANCOVA.

For linear trends, an appropriate regression trend model would be a linear trend either without:

$$Y_t = \beta_0 + \beta_1 \mathrm{DATE} + e_t$$

or, with explanatory variables as appropriate:

$$Y_t = \beta_0 + \beta_1 \mathrm{DATE} + \Sigma\beta_i X_i + e_t$$

Where: Y_t = Water quality variable value at time *t*. If Y is log normal, then Y_t is the log-transformed water quality variable value.

X_i = Explanatory variable, i=2,3… (X_2, X_3, etc. could also be log-transformed; the DATE variable is considered X1)

β_0 = Intercept

β_1 = Slope or linear trend on DATE

β_i = Regression coefficients for explanatory variables

e_t = Error term (this is denoted as V_t if the error series has an autocorrelated structure; see Step 4 and Example 1)

Note that even though no (zero) trend is expected if this test uses only the pre-BMP data, it is appropriate to include the trend (DATE) term in the statistical model when this is the planned statistical model.

For a step trend, the DATE can have the values of 0 for pre-BMP or 1 for post-BMP data. When planning or evaluating a monitoring design, there may not yet be any post-BMP data and only pre-BMP data would then be used in the MDC calculations.

Note: the paired-watershed study and the above/below-before/after watershed designs are analyzed using an ANCOVA where 'Date' is 0 or 1 and the explanatory variable is either the control watershed values (concentrations/loads) or the upstream values paired with the treatment or downstream values, respectively.

Step 2. ***Perform Exploratory Data Analyses.*** Preliminary data inspections are performed to determine if the residuals are distributed with a normal distribution and constant variance. Normal distribution is required in the parametric analyses; constant variance is required in both parametric and nonparametric analyses. The water quality monitoring data are usually not normal, however, and often do not exhibit constant variance over the data range.

Exploratory data analysis (*Meals and Dressing 2005*) is an important step in determining whether available data meet the assumptions (e.g., normality, constant variance) of planned statistical tests.

The water quality data sets are examined using univariate procedures such as those available with the SAS procedure PROC UNIVARIATE or within JMP (SAS Institute 2010, 2008) to verify distributional assumptions required for statistical procedures. Specific attention is given to the statistics on normality, skewness, and kurtosis. Both the original and logarithmic transformed values are tested.

Step 3. ***Perform Data Transformations.*** Water quality data typically follow log-normal distributions and the base 10 logarithmic transformation is typically used to minimize the violation of the assumptions of normality and constant variance. In this case, the MDC calculations use the log-transformed data until the last step of expressing the percent change. Alternatively, the natural log transformation may be used.

The logarithmic base 10 transformation applies to all dependent water quality variables used in trend detection (i.e., suspended sediment, TP, ortho phosphorus, and fecal coliform). Technically, explanatory variables in statistical trend models do not have any distributional requirements because it is only the distribution of the residuals that is crucial. However, if they do exhibit log normal distributions, explanatory variables are also log-transformed which usually helps with the distribution requirements of the residuals. Typical explanatory variables that are log-transformed include upstream concentrations and stream flow.

Step 4. ***Test for Autocorrelation.*** Tests are performed on the water quality time series to determine if there is autocorrelation. An autoregressive, lag 1 (AR(1)) error structure (i.e., correlation between two sequential observations) in the water quality trend data is common. The tests usually assume samples are collected with equal time intervals. The regression trend models used are the same as those planned for the future trend analyses (See Step 1). The data should be ordered by collection date.

The Durbin Watson (DW) test for autocorrelation can be performed on the residuals from the linear regression models to determine if the concentration measurements are related to previous measurements. This test can be performed with the SAS procedure PROC REG or PROC AUTOREG (SAS Institute, 1999), or within the least squares regression analysis of JMP. The Durbin Watson test assumes the residuals exhibit an AR(1) autocorrelation structure. Alternatively, the significance of the first order autocorrelation coefficient is tested in SAS using a time series statistical procedure such as PROC AUTOREG or time series analyses within JMP. It should be noted that PROC AUTOREG allows for missing Y-values, but equally-spaced date entries should all be included in the data set.

> Appropriate statistics software packages can make the job of MDC analysis a lot easier, but it is important to not treat these packages as black boxes.

Alternatively, the assumption of independent residuals can be tested by passing the residuals from these regression trend models to the SAS procedure PROC ARIMA

(SAS Institute, 1999) or time series analysis within JMP (SAS Institute, 2008). The autocorrelation structure is examined to determine if the independence assumption is valid and, if not, to determine the appropriate autocorrelation structure for the simple trend models. The chi-square test of white noise supplied by PROC ARIMA is also used to test whether the residuals are independent.

Step 5. ***Calculate the Estimated Standard Error.*** The variability observed in either historic or pre-BMP water quality monitoring data is used to estimate the MDC. Any available post-BMP data can also be included in this step. The estimated standard error is obtained by running the same statistical model that will be used to detect a trend once BMPs have been installed (same trend models identified in Step 1).

For a linear trend, an estimate of the **standard deviation on the slope** over time is obtained by using the output from statistical regression analysis with a linear trend, time series errors (if applicable), and appropriate explanatory variables. If the planned monitoring timeframe will be longer than that from which the existing data were obtained, the standard deviation on the future slope can be estimated by:

$$s_b = s'_b \sqrt{\frac{(n - 2)}{(C * n - 2)}}$$

Where: s_b = estimate for the standard deviation of the trend for the total planned duration of monitoring

s'_b = standard deviation of the slope for the existing data

n = number of samples in the existing data

C = correction factor equal to the proportional increase in planned samples. For example, if 4 years of existing data are available and 8 years of total monitoring is planned, C=2 (i.e., 8/4). This factor will reduce the standard error on the slope and, therefore, the amount of change per year required for statistical significance.

A large sample approximation for the adjustment factor is:

$$s_b = s'_b \sqrt{\frac{1}{C}}$$

For a step trend, it is necessary to have an estimate of the **standard deviation of the difference between the mean values of the pre-BMP vs. post-BMP data** ($s_{(\bar{X}pre-\bar{X}post)}$). In practice, an estimate is obtained by using the following formula:

$$s_{(\bar{X}pre-\bar{X}post)} = \sqrt{\frac{MSE}{n_{pre}} + \frac{MSE}{n_{post}}}$$

Where: $s_{(\bar{X}pre-\bar{X}post)}$ = estimated standard error of the difference between the mean values of the pre- and the post-BMP periods.

MSE = s_p^2 = Estimate of the pooled Mean Square Error (MSE) or, equivalently, variance (s_p^2) within each period. The MSE estimate is obtained from the output of a statistical analysis using a *t*-test or ANCOVA with appropriate time series and explanatory variables.

The variance (square of the standard deviation) of pre-BMP data can be used to estimate MSE or s_p^2 for both pre- and post-BMP periods if post-BMP data are not available and there are no explanatory variables or autocorrelation (see Example 2). For log normal data calculate this value on the log-transformed data.

Missing values are allowed. It is not important here that no trend is present because this step obtains the estimate on the standard deviation of the trend statistic.

For both linear and step trends, if autocorrelation is present a time series statistical procedure such as SAS's PROC AUTOREG that uses Generalized Least Squares (GLS) with Yule Walker methods should be employed because it takes into account the autocorrelation structure of the residuals to obtain valid standard deviations (Brocklebank and Dickey, 1986). The standard error on the trend estimate for simple trend models (e.g., step, linear, or ramp trends) with AR(1) error terms is **larger** than that (incorrectly) calculated by Ordinary Least Squares (OLS). Matalas (1967) cited theoretical adjustments that can be used. The true standard deviation has the following large sample approximation:

For projects in the planning phase it is possible to estimate MSE using only pre-BMP data or data from nearby and similar watersheds. The MDC estimates from such approaches, however, are likely to be less reliable than those made using datasets from the study watershed with appropriate explanatory variables and multiple years of data.

$$s_b = s'_b \sqrt{\frac{1+\rho}{1-\rho}}$$

Where: s_b = true standard deviation of the trend (slope or difference between 2 means) estimate (e.g., calculated using GLS)

s'_b = incorrect variance of the trend estimate calculated without regard to autocorrelation using OLS (e.g., using a statistical linear regression procedure that does not take into account autocorrelation)

ρ = autocorrelation coefficient for autoregressive lag 1, AR(1)

Step 6. ***Calculate the MDC.*** MDC is essentially one-half of the confidence interval for the slope of a linear regression model or for the difference between the mean values of the pre- and post-BMP periods.

For a **linear trend**, the MDC is calculated by multiplying the **estimated standard deviation of the slope** by the *t*-statistic and the total monitoring timeframe:

$$\mathbf{MDC = (N) * } t_{(n*N-2)df} \mathbf{* 365 *} s_{b1}$$

Where: $t_{(n*N-2)df}$ = One-sided Student's *t*-statistic (α=.05)

N = Number of monitoring years

n = Number of samples per year

df = degrees of freedom

365 = Correction factor to put the slope on an annual basis when DATE is entered as a Date (day) variable, e.g., the slope is in units per day. If DATE values were 1-12 for months and the slope was expressed 'per month' then this value would be "12."

s_{b1} = Standard deviation on the slope estimated for the total expected monitoring duration (from Step 5)

MDC = the MDC on either the original data scale or the log scale if the data were log-transformed

For a **step trend**, the MDC is one-half of the confidence interval for detecting a change between the mean values in the pre- vs. post-BMP periods.

$$\mathrm{MDC} = t_{(n_{pre} + n_{post} - 2)} * s_{(\bar{X}_{pre} - \bar{X}_{post})}$$

In practice, an estimate is obtained by using the following equivalent formula:

$$\mathrm{MDC} = t_{(n_{pre} + n_{post} - 2)} \sqrt{\frac{\mathrm{MSE}}{n_{pre}} + \frac{\mathrm{MSE}}{n_{post}}}$$

Where: $t_{(n_{pre} + n_{post} - 2)}$ = one-sided Student's *t*-value with $(n_{pre} + n_{post} - 2)$ degrees of freedom.

$n_{pre} + n_{post}$ = the combined number of samples in the pre- and post-BMP periods

$s_{(\bar{X}_{pre} - \bar{X}_{post})}$ = estimated standard error of the difference between the mean values in the pre- and the post-BMP periods.

MSE = s_p^2 = Estimate of the pooled Mean Square Error (MSE) or, equivalently, variance (s_p^2) within each period. The MSE estimate is obtained from the output of a statistical analysis using a *t*-test or ANCOVA with appropriate time series and explanatory variables. If post-BMP data are not available, no autocorrelation is present, and no explanatory variables are appropriate, MSE or s_p^2 can be estimated by the variance (square of the standard deviation) of pre-BMP data.

The pre- and post-BMP periods can have different sample sizes but should have the same sampling frequency (e.g., weekly).

The following considerations should be noted:

- The choice of one- or two-sided *t*-statistic is based upon the question being asked. Typically, the question is whether there has been a statistically significant decrease in pollutant loads or concentrations and a one-sided *t*-statistic would be appropriate. A two-sided *t*-statistic would be appropriate if the question being evaluated is whether a change in pollutant loads or concentrations has occurred. The value of the *t*-statistic for a two-sided test is larger, resulting in a larger MDC value.
- At this stage in the analysis, the MDC is either in the original data scale (e.g., mg/L) if non-transformed data are used, or, more typically in the log scale if log-transformed data are used.

Step 7. ***Express MDC as a Percent Decrease.*** If the data analyzed were not transformed, MDC as a percent change (MDC%) is simply the MDC from Step 6 divided by the average value in the pre-BMP period expressed as a percentage (i.e., MDC% = 100*(MDC/mean of pre-BMP data)).

When calculating MDC as a percent change it is important to note whether the data analyzed were log-transformed because the formula is different from that used for data that were not log-transformed.

If the data were **log-transformed**, a simple calculation can be performed to express the MDC as a percent decrease in the geometric mean concentration relative to the initial geometric mean concentration or load. The calculation is:

$$\mathbf{MDC\% = (1 - 10^{-MDC}) * 100}$$

Where: MDC is on the log scale and MDC% is a percentage.

For log-transformed data MDC is the difference required on the logarithmic scale to detect a significant decreasing trend (calculated in Steps 5 and 6 using log-transformed data). MDC% and MDC are positive numbers if mean concentrations decrease over time. For example, for MDC= 0.1 ($10^{-0.1} = 0.79$), the MDC% or percent reduction in water quality required for statistical significance = 21%; for MDC = 0.2 ($10^{-0.2} = 0.63$), MDC% = 37%. In the cases where detection of a positive trend is desired (e.g., Secchi depth measurements), the percent decrease would be negative and the input for MDC must be forced to be negative.

It should be noted that if the natural logarithmic transformation had been used, then:

$$MDC\% = (1 - \exp^{-MDC}) * 100$$

Examples

Example 1. ***A linear trend with autocorrelation and covariates or explanatory variables; Y values log-transformed.*** The basic statistical trend model used in this example is linear regression with time series errors, techniques documented by Brocklebank and Dickey (1986). Typically, Autoregressive Lag 1 or AR(1) is appropriate and a DATE explanatory variable is included in the model. The DATE variable is used to estimate the magnitude of a linear trend and to estimate the variation not accounted for by the linear trend term observed in the water quality measurements. The estimate of variation on the "slope" of DATE is then used to calculate an estimate of Minimum Detectable Change (MDC). The significance of the linear trend, its magnitude, or its direction is not important in the calculation of MDC. The important statistical parameter is the **standard deviation on the slope estimate** of the linear trend.

The SAS procedure, PROC AUTOREG (SAS Institute, 1999) can be used in this analysis. The linear regression model estimated at each monitoring location is:

$$Y_t = \beta_0 + \beta_1 DATE + V_t$$

or, with explanatory variables:

$$Y_t = \beta_0 + \beta_1 DATE + \Sigma \beta_i X_i + V_t$$

Where: Y_t = Log-transformed water quality variable value at time t,
V_t = Error term assumed to be generated by an autoregressive process of order 1, AR(1).
β_0 = Intercept
β_1 = Slope or linear trend on DATE
β_i = Unique regression coefficients for each explanatory variable
X_i = Explanatory variable, i=2,3,..,

The standard deviations on the slope over time from linear regression models are used to calculate the MDCs. A significance level of α = .05 and a Type II error of b=0.5 are assumed. The standard deviation on the slope is a function of the mean square error (MSE or s^2) estimated by the Yule Walker Method and Generalized Least Squares, degree of autocorrelation, and the degrees of freedom (d.f.). The d.f. is a function of the number of monitoring years and sample frequency. If continued sampling is planned, the estimate of the standard deviation of the trend slope is adjusted by a correction factor given in Step 5.

MDC is calculated by:

$$\mathbf{MDC = (N) * t_{(n*N-2)df} * 365 * s_{b1}}$$

Where: $t_{(n*N-2)df}$ = One-sided Student's t-statistic (α = .05)

N = Number of monitoring years

n = Number of samples per year

365 = Correction factor to put the slope on an annual basis because DATE is assumed to be entered as a Date variable (i.e., the slope is in units per day). If DATE values were entered as 1–12 for months causing the slope to be expressed as 'per month' then this value would be "12."

s_{b1} = Standard deviation on the slope

MDC = MDC on the log scale in this case

The calculations are illustrated below with the following assumptions:

N = 5 years existing (10 years planned)
n = 52 weekly samples per year

DATE was entered into the computer program as a DATE, so the slope is expressed in units per day

$t_{(n*N-2)df} = t_{258}$ = 1.6513 (one-sided)

s_{b1} = 0.0000229 (This is the standard deviation on the slope for the trend, which is log scale for this example because log-transformed data are assumed. It is very important to carry several significant digits because the number might be small.)

The MDC for the existing 5 years of data can be calculated as follows. The calculations for MDC and then MDC% for this example using Y values that are log-transformed are:

$MDC = (N) * t_{(n*N-2)df} * 365 * s_{b1}$
MDC = 5 * 1.6513 * 365 * 0.0000229
MDC = 0.06901 (units on log scale)
MDC% = $(1 - 10^{-MDC})$ * 100 (percentage on geometric mean)
MDC% = $(1 - 10^{-0.06901})$ * 100
MDC% = 15% (percentage on geometric mean) or an average of 3% change per year

Note: If a 2-sided t-statistic value was used then t=1.969, MDC (log scale) is 0.0823, and MDC% is 17%.

The MDC estimate if the sampling duration will be doubled to a total of 10 years:

$$s_{b1(10\ years)} = s'_{b1(5\ years)} \sqrt{\frac{(n - 2)}{(C * n - 2)}} = 0.0000229 \sqrt{\frac{(260 - 2)}{(2 * 260 - 2)}}$$

$$= 0.0000229 * 0.70574$$

$$= 0.00001616$$

MDC (10 years) = 10 * 1.6513 * 365 * 0.00001616

= 0.0974 (units on log scale)

= 20% over 10 years (or an average of 2% change per year)

The addition of explanatory variables can decrease the magnitude of the MDC.

The addition of appropriate explanatory variables and sampling frequency can decrease the magnitude of the calculated MDC. For example, Spooner et al. (1987) demonstrated that adding salinity as a covariate in the Tillamook Bay, Oregon watershed study decreased the MDC% for fecal coliform over an 11-year period of time (with biweekly samples) from 42% to 36%. For the same study, the MDC% for fecal coliform decreased from 55% to 42% when comparing monthly to biweekly sampling over an 11-year study. Spooner et al. (1987 and 1988) also demonstrated that variability and therefore MDC is also affected by the pollutant measured, the size of the watershed, and appropriate selection of explanatory variables.

Example 2. ***A step trend, no autocorrelation, and no covariates or explanatory variables; Y values on original scale (not transformed).*** In this example, the plan would be to detect a significant change in the average values between the pre- and post-BMP periods. The pre- and post-BMP periods can have different sample sizes but should have the same sampling frequency (e.g., weekly).

In this simplified situation, the MDC would be equivalent to the Least Significant Difference (LSD). MDC would be calculated as:

$$MDC = t_{(n_{pre} + n_{post} - 2)} \sqrt{\frac{MSE}{n_{pre}} + \frac{MSE}{n_{post}}}$$

Where: $t_{(n_{pre} + n_{post} - 2)}$ = one-sided Student's t-value with $(n_{pre} + n_{post} - 2)$ degrees of freedom.

$n_{pre} + n_{post}$ = the combined number of samples in the pre- and post-BMP periods

MSE = Estimate of the pooled Mean Square Error (MSE) or variance (s_p^2) within each period. The variance (square of the standard deviation) of pre-BMP data can be used to estimate MSE or s_p^2 for both pre- and post-BMP periods if post-BMP data are not available (the usual case when designing monitoring programs). For log normal data calculate this value on the log-transformed data.

The calculations are illustrated below with the following assumptions:

n_{pre} = 52 samples in the pre-BMP period
n_{post} = 52 samples in the post-BMP period
Mean X = 36.9 mg/l, mean of the 52 samples in the pre-BMP period
s_p = 21.2 mg/L = standard deviation of the 52 pre-BMP samples
MSE = s_p^2 = 449.44
$t_{(n_{pre} + n_{post} - 2)} = t_{102} = 1.6599$

The MDC would be:

$$MDC = t_{(n_{pre} + n_{post} - 2)} \sqrt{\frac{MSE}{n_{pre}} + \frac{MSE}{n_{post}}}$$

$$MDC = 1.6599 \sqrt{\frac{449}{52} + \frac{449}{52}}$$

MDC = 6.9 mg/l

Percent change required = MDC% = 100*(6.9/36.9) = 19%.

Use the equation described under "Step 7" above to calculate percent change for log-transformed data. If the data are autorcorrelated, use a time series model, or the approximation given in Step 5 to adjust the standard error of the difference in the pre- and post-BMP means.

Example 3. ***Paired-watershed study or Above/Below-Before/After watershed study analyzed using Analysis of Covariance (ANCOVA); Y values log-transformed; no autocorrelation.*** The paired-watershed approach requires a minimum of two watersheds, control and treatment, and two periods of study, calibration and treatment (Clausen and Spooner, 1993). The control watershed accounts for year-to-year or seasonal climatic variations. During the calibration period, the two watersheds are treated identically and paired water quality data are collected (e.g., event-based, weekly). During the treatment period, the treatment watershed is treated with a BMP(s) while the control watershed remains under the same management employed during the calibration period. Under the **above/below-before/after** approach water quality downstream and upstream of a BMP location is monitored for time periods before and after BMP implementation.

Data from these two watershed designs can be analyzed with similar **ANCOVA** approaches. The Y values in the equation below are taken from either the treatment watershed in a paired-watershed study or the downstream site in an above/below study. The values for the explanatory (X) variable are taken from the control watershed in a paired-watershed design or from the upstream site in an above/below design. Each monitoring design has another explanatory variable that is represented by 0 or 1 for the 'pre-BMP' and 'post-BMP' periods, respectively.

The ANCOVA model is:

$$Y_t = \beta_0 + \beta_1(\text{Period}) + \beta_2 X_t + e_t$$

Where: Y_t = Water quality variable value at time *t* (from treatment watershed or downstream site). If Y is log normal, then Y_t is the log-transformed water quality variable value.

Period = '0' for pre-BMP period and '1' for post-BMP period (alternatively, period can be treated as a grouping variable and entered as characters).

X_t = Explanatory variable value at time t (water quality values from control watershed or upstream site). Values are log-transformed if distribution is log-normal.

β_0 = Y intercept

β_1, β_2 = Regression coefficients

e_t = Error term

The SAS procedure PROC GLM (SAS Institute, 2010), JMP (SAS Institute, 2008), or SPSS (IBM, 2011) can be used for the analysis. Period would be identified as a 'Class' variable in PROC GLM or 'Character' variable in JMP. The "Fit Model" dialog box would be used in JMP. Users would select the Y variable, use the "Add" option to include the X (i.e., control) and Period variables, and then choose 'Run Model.'

It is important to note that because MDCs are generally calculated prior to the treatment period, this example assumes that the slopes for the pre- and post-BMP periods will be similar. The Durbin Watson statistic to check for autocorrelation can be calculated as an option under both SPSS and either SAS procedure. If autocorrelation is significant, PROC AUTOREG can be used for the analysis with Period values set to numeric '0' and '1'.

The treatment effect will be the difference in the least square means (lsmeans) between the pre- and post-BMP periods. The MDC is the difference that would be statistically significant and therefore based upon the standard error of the difference between lsmeans values. The lsmeans are the estimates of the values of Y for the pre- and post-BMP periods evaluated at the overall average value of all the X (treatment) values collected during the entire study period. MDC is calculated from the standard error on the difference in lsmeans. The standard error is given by the JMP procedure when users choose the option for 'detailed comparisons'.

The MDC on the log values would be:

$$\mathbf{MDC} = t_{(n_{pre} + n_{post} - 3)} * s_{(\text{lsmean}_{pre} - \text{lsmean}_{post})}$$

Where: $t_{(n_{pre} + n_{post} - 3)}$ = One-sided Student's t-value with $(\mathbf{n}_{pre} + \mathbf{n}_{post} - 3)$ degrees of freedom (Note that the t-statistic given in JMP is the two-sided value).

$\mathbf{n}_{pre} + \mathbf{n}_{post}$ = The combined number of samples in the pre- and post-BMP periods

$s_{(\text{lsmean}_{pre} - \text{lsmean}_{post})}$ = Estimated standard error of the difference between the least square mean values in the pre- and the post-BMP periods. This is computed by using the following approximation (adapted from Snedecor and Cochran, 1967, p. 423):

$$\sqrt{\mathbf{MSE} * \frac{2}{\mathbf{n}} * \mathit{Factor}}$$

MSE is found in the Analysis of Variance table from the output of the applied statistical analysis, and n is the number of samples within each period. The adjustment "Factor" is 1 or greater and increases when the difference between the mean of the X (control watershed or upstream) data in the pre-BMP period compared to the post-BMP period increases. It is assumed to be "1" for MDC calculations. This "Factor" adjustment makes clear the importance of collecting samples in the pre-BMP and post-BMP periods that have similar ranges and variability in hydrological conditions.

To express MDC as a percentage change required in geometric mean value:

$$\mathbf{MDC\%} = (1 - 10^{-\mathrm{MDC}}) * 100\text{, where MDC is on the log scale}$$

Summary

The Minimum Detectable Change is the minimum change in a pollutant concentration (or load) over a given period of time required to be considered statistically significant. MDC calculations can be very helpful in the design of cost-effective monitoring programs, as well as increasing awareness regarding the potential a watershed project has for achieving measurable results. These calculations also illustrate the value of adjusting for changes in hydrologic and meteorological variables. Not only is the ability to detect real changes increased, but valid conclusions regarding the magnitude and direction of measured change(s) in a water quality variable can be made. Calculation of MDC can also be used to illustrate the importance of relatively long monitoring time frames. In addition, comparison of the actual changes in water quality to the MDC values can be used to document BMP effectiveness on a subwatershed basis.

The magnitude of MDC is often larger than expected by watershed projects and funding agencies, leading to misunderstanding regarding the needed level of BMP implementation, intensity of monitoring, and duration of monitoring. The magnitude of MDC can be reduced by:

- Accounting for changes in discharge, precipitation, ground water table depth, or other applicable hydrologic/meteorological explanatory variable(s).
- Accounting for changes in incoming pollutant concentrations upstream of the BMP implementation subwatershed (i.e., upstream concentrations).
- Increasing the length of the monitoring period.
- Increasing the sample frequency.
- Applying the statistical trend technique that best matches the implementation of BMPs and other land use changes.

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